

1. Prove the following asymptotic bounds. (40 marks)
 - a. $60n^2 + 5n + 1 \in O(n^2)$. (10 marks)
 - b. $1 + 2 + \dots + n \in \Omega(n^2)$. (10 marks)
 - c. $2n + 3 \lg n \in \Theta(n)$. (10 marks)
 - d. $\lg^3 n \in o(n^{0.5})$. (10 marks)

2. Give asymptotic upper bound for $T(n)$ in each of the following recursions. Make your bound as tight as possible, and justify your answers. Asymptotic bound is given without justification will receive only half marks. (30 marks)
 - a. $T(n) = 3T(\lfloor n/3 \rfloor) + n/2$. (10 marks)
 - b. $T(n) = T(n/2) + 2T(n/4) + n$. (10 marks)
 - c. $T(n) = \sqrt{n}T(\sqrt{n}) + n$. (hint: use changing variables with recursion tree) (10 marks)

3. Given the following deterministic insertion sort algorithm, answer the following questions. (30 marks)

```

void insertionsort (int n, keytype S[])
{
    index i, j, key;
    for (j=2; j<=n; j++){
        key = S[j];
        i = j - 1;
        while (i>0 && S[i]>key){
            S[i+1] = S[i];
            i = i - 1;
        }
        S[i+1] = key;
    }
}

```

- a. Please write the pseudocode of the corresponding randomized version of this insertion sort algorithm. (10 marks)
- b. Analyze the average running time of the randomized algorithm by probabilistic analysis. (20 marks)