1. Recall the Hamiltonian circuits problem: find a path that starts at a given vertex, visits each vertex in the graph exactly once, and ends at the starting vertex.
   (a) Describe its corresponding decision problem. (10 marks)
   (b) Given an answer of the Hamiltonian circuits problem $vindex$, which is an array where each item stores a vertex index of the path, and the affinity matrix W, write the pseudocode of the verification algorithm for its corresponding decision problem. (20 marks)
   (c) Prove that the verification algorithm is in polynomial time. (10 marks)

2. For the set-cover problem, given
$$X = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9\},$$
and
$$\begin{aligned} F = \{&S_1 = \{e_1, e_2, e_3\}, \\ &S_2 = \{e_2, e_3, e_5, e_6, e_7, e_9\}, \\ &S_3 = \{e_1, e_2, e_5\}, \\ &S_4 = \{e_3, e_4, e_7\}, \\ &S_5 = \{e_5, e_6, e_8, e_9\}, \\ &S_6 = \{e_3, e_5, e_6, e_7, e_8\}\}, \end{aligned}$$
apply the greedy_set_cover algorithm to compute a set of S_i which covers all elements in X and show each step of your computation. Is it optimal? If NOT, show the optimal solution and the actual performance ratio in this case. (30 marks)

3. For MAX-CNF satisfiability problem, we set the length of each clause is at least 3, namely $l_j \geq 3$ for all j.
   (a) What is the approximation ratio $\rho(n)$ if the same randomized algorithm is adopted? Show the result with calculation procedure. (15 marks)
   (b) Assume now $l_j \geq k$ for all j. As the k increases, will the randomized algorithm perform better or worse? Show your analysis with calculation procedure. (15 marks)